ANSWER TO I_2.

~✓ $i : I \vdash A(i) : \mathrm{Type}$ or $A : I \to \mathrm{Type}$

$$\frac{I : \mathrm{Type} \qquad \boxed{(\forall i : I)(A(i) : \mathrm{Type})}}{\prod_{i:I} A(i) : \mathrm{Type}}\ \Pi\text{-F}$$

$i : I \vdash a(i) : A(i)$

$$\frac{\boxed{(\forall i : I)(\alpha(i) : A(i))}}{\lambda(i:I).\alpha(i) : \prod_{i:I} A(i)}\ \Pi\text{-I}$$ ✓

$i : I$

$$\frac{f : \prod_{i:I} A(i)}{\boxed{(\forall i : I)(f(i) : A(i))}}\ \Pi\text{-E}$$

weird choice of letter
(argument and parameter look the same)

β: $(\lambda(i:I).\alpha(i))\,\boxed{i} = \alpha(\boxed{i})$

Also handy: substitution notation
$(\lambda i.a)\,t = a[i := t]$
or $a[t/i]$

η: $f = \lambda(i:I).f\,i$ ✓

ANSWER TO G_1. ✓

bad name for an element of γ

We need to show the following, for all $\alpha, \beta : \mathrm{Type}$ and $f : \gamma \to \alpha$, $g : \gamma \to \beta$:

$$\begin{tikzcd} & \alpha \times \beta \arrow[dl, "outl"'] \arrow[dr, "outr"] & \\ \alpha & & \beta \\ & \gamma \arrow[ul, "f"] \arrow[ur, "g"'] \arrow[uu, dashed, "!h"] & \end{tikzcd}$$

We define $h \stackrel{def}{=} \lambda \boxed{\gamma}.\langle f\gamma, g\gamma\rangle : \gamma \to \alpha \times \beta$.
Then, the diagram commutes: $outl\langle f\gamma, g\gamma\rangle = f\gamma$, $outr\langle f\gamma, g\gamma\rangle = g\gamma$.

To show h is unique, assume some $h' : \gamma \to \alpha \times \beta$ has the same property.
Then, $h'\gamma = \langle outl\, h'\gamma, outr\, h'\gamma\rangle$
$= \langle f(\gamma), g(\gamma)\rangle$
$= h\gamma$.

Therefore, by the extensionality of functions, $h' = h$.

ANSWER TO P1.

suppose $\neg(P\vee\neg P)$
claim $P\vee\neg P$
  $\boxed{\text{case-r}}$ which disjunction are you using?
  Ah! You meant: « (choose/go) Right. »
  suppose P
  claim $P\vee\neg P$
    $\boxed{\text{case-l}}$ (same): Left.
    exact
  apply $\neg(P\vee\neg P)$ to $P\vee\neg P$
apply $\neg(P\vee\neg P)$ to $P\vee\neg P$

ANSWER TO D1.

~~We need to show $a\vee(b\wedge c)=(a\vee b)\wedge(a\vee c)$ and $a\wedge(b\vee c)=(a\wedge b)\vee(a\wedge c)$ We know that $a\vee(b\wedge c)\le(a\vee b)\wedge(a\vee c)$ and $(a\wedge b)\vee(a\wedge c)\le a\wedge(b\vee c)$ always hold.~~

$$\dfrac{\dfrac{\dfrac{}{b\le b\vee d}\qquad\dfrac{}{b\le a}}{b\le(b\vee d)\wedge a}\qquad\qquad\dfrac{\dfrac{}{d\le b\vee d}\qquad\dfrac{}{a\le a}}{d\wedge a\le(b\vee d)\wedge a}}{b\vee(d\wedge a)\le(b\vee d)\wedge a}$$

labeling your inference rules is a good idea in many cases (this is one of them) ü

ANSWER TO $T4$.

Binary case $(+)$: Αν έχουμε δύο set D^0 και D^1, κάθε στοιχείο του Π? $\boxed{?}$ αποτελεί μια διάδα $\langle D^0, D^1 \rangle$, δηλαδή σχηματίζει το $(\times)$. ~

Con~~tinuous~~stant case $(\rightarrow)$: Αν έχουμε πάντα το ίδιο set D σε κάθε index $i \in I$, τότε κάθε στοιχείο του Π ~~σχηματίζει~~ είναι συνάρτηση $f: I \rightarrow D$

ποιανού Π;

πώς το χρησιμοποιείς;

outl := ?

outr := ?

$\langle \_, \_ \rangle$:= ?

(Θα χρειαστείς ένα 2-element-type (2) για να παίξει το ρόλο του index type.)

ANSWER TO $O1$.

$$\dfrac{\dfrac{\dfrac{}{b \vee d \geq b} \qquad a \geq b}{(b \vee d) \wedge a \leq b} \qquad \dfrac{\dfrac{}{d \wedge a \leq a} \qquad a \geq b}{b \vee (d \wedge a) \geq a}}{b \vee (d \wedge a) \leq (b \vee d) \wedge a}$$

X ?

this branch looks like you are using

this branch look like you are using

Answer to T1.

Για (×):

$$\frac{A : type \quad B : type}{A \times B : type} \ (\times)\text{-F}$$

$$\frac{a : \alpha \quad b : \beta}{\langle a, b \rangle : \alpha \times \beta} \ (\times)\text{-I}$$

$$\frac{w : \alpha \times \beta}{w.l : \alpha} \ (\times)\text{-E}_l \qquad \frac{w : \alpha \times \beta}{w.r : \beta} \ (\times)\text{-E}_r$$

β: $\langle a, b \rangle . l = a$

$\langle a, b \rangle . r = b$

η: $w = \langle w.l, w.r \rangle$

Answer to P2.

P decidable $\Longrightarrow$ P stable:

$P \vee \neg P \vdash \neg\neg P \Rightarrow P$

Suppose $\neg\neg P$. -- $P \vee \neg P, \neg\neg P \vdash P$

Sep. $P \vee \neg P$ by cases.

Case P: -- $P, \neg\neg P \vdash P$

Exactly P.

Case $\neg P$: -- $\neg P, \neg\neg P \vdash P$

Contradiction.

$\square$

ANSWER TO P2.

$\vdash p \vee \neg p \Rightarrow (\neg\neg p \Rightarrow p)$

Suppose $p \vee \neg p$.

Seperate by cases for "v"..

χωρίς κενό εκλίψα

Case-L : p

Suppose $\neg\neg p$. -- $p \vee \neg p, p, \neg\neg p \vdash p$

Exact(ly) p.

Case-R : $\neg p$

Suppose $\neg\neg p$. -- $p \vee \neg p, \neg p, \neg\neg p \vdash p$

Boom.

Αποδείξαμε ότι

P decidable $\Rightarrow$ p stable

η άλλη συνεπαγωγή είναι αδύνατο να αποδειχθεί χωρίς την χρήση του νόμου LEM ή LDN (why risk this claim?)

bonus HW: τι πρέπει να αποδείξουμε για αυτό;

ANSWER TO ~~10~~ T1

η of (x)

$w = \langle a, b \rangle$ $\lambda w.\langle w.\ell, w.r \rangle$

revise!

ANSWER TO O_1~.

$$\frac{d\wedge a\le d \qquad d\le b\vee d}{d\wedge a\le b\vee d}\ \le\text{-trans}$$

$d\le d$, $d\wedge a$ lb of d, $b\vee d$ upb of d

$$\dfrac{\dfrac{\dfrac{\cancel{b\le b} \qquad \cancel{b\vee d} \text{ upper bound of } b}{b\le b\vee d}\ {\scriptstyle +\text{trans}}\ {\scriptstyle v\text{-ub-L}} \qquad \dfrac{\boxed{}}{d\wedge a\le b\vee d}\ {\scriptstyle +\text{trans}}}{b\vee(d\wedge a)\le(b\vee d)} \qquad \dfrac{\dfrac{}{b\le a}\ \text{Givens} \qquad \dfrac{a\cancel{\le a} \qquad d\cancel{\wedge} a\ \cancel{\text{lb of}}\ a}{d\wedge a\le a}\ {\scriptstyle +\text{trans}}\ \wedge\text{-lb-R}}{b\vee(d\wedge a)\le a}}{b\vee(d\wedge a)\le(b\vee d)\wedge a}$$

ANSWER TO C_3~.

$(\exists f')[f'f=1\ \&\ ff'=1]$

Let $f:A\to B$ morphism, A, B obj of a cat C

f split mono $\wedge$ epi $\Longrightarrow$ f iso $\dashv (\exists r)(\exists s): r\circ f=1_A, f\circ s=1_B$

~~disjunction~~ con (but actually it is a $(\exists)$)

Ext-~~R~~ L for $\wedge$..

$\boxed{\text{Let } r:B\to A \text{ such that } r\circ f=1_A}$

~~Use~~

$\boxed{\text{Split}}$

Left.

$\boxed{\text{use } r}$ ×

Exactly $r\circ f=1$.

Split. ✓

Right,

use $1_A\circ r$ ×

---. $\vdash$ for $=1$

$$\dfrac{\dfrac{rf=1}{frf=1f}}{fr=1}\ f \text{ epi}$$

Ext-L for $\wedge$

Compute $1_B\circ f\circ 1_A\circ r = \ldots = r\circ f\circ 1_B$?

Thence $f\circ(1_A\circ r)=1_B$

P decidable $\iff$ $P \vee \neg P$

stable $\iff$ $\neg\neg P \supset P$

$P \vee \neg P \Rightarrow \neg\neg P \supset P$

you could "factor out" this 'suppose $\neg\neg P$' line.

Suppose $P \vee \neg P$ -- $P \vee \neg P \vdash \neg\neg P \supset P$

Separate by cases

CASE-L

Suppose $\neg\neg P$ -- $P \vdash \neg\neg P \supset P$

Exactly P -- $P, \neg\neg P \vdash P$

CASE-R

Suppose $\neg\neg P$ -- $\neg P \vdash \neg\neg P \supset P$

Apply $\neg\neg P$ to $\neg P$ -- $\neg P, \neg\neg P \vdash P$ $P \Rightarrow \bot, (P \Rightarrow \bot) \Rightarrow \bot \vdash P$

Boom