P(r)oset

lec01
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Poset carrier

$\mathcal{P} \equiv (P\ ;\ \leq)$

$P \quad : \quad \text{Set}\ \alpha$

$(\leq) \quad : \quad \underbrace{P \times P \to \text{Prop}}_{\text{BinRel}(P)}$

$\text{Set} \quad : \quad \text{Type} \to \text{Type}$

$\text{Set Int},\quad \text{Set Person},\quad \text{Set Nat} \quad : \quad \text{Type}$

Proset

pré-order

$(\leq)$-refl	$a \leq a$	$\dfrac{}{a \leq a}$ refl
$(\leq)$-trans	$a \leq b \ \&\ b \leq c \Rightarrow a \leq c$	$\dfrac{a \leq b \quad b \leq c}{a \leq c}$ trans
$(\leq)$-antis	$a \leq b \ \&\ b \leq a \Rightarrow a = b$	$\dfrac{a \leq b \quad b \leq a}{a = b}$ sym

Arr(Poset)

$$\mathcal{P} \xrightarrow{f} \mathcal{Q}$$

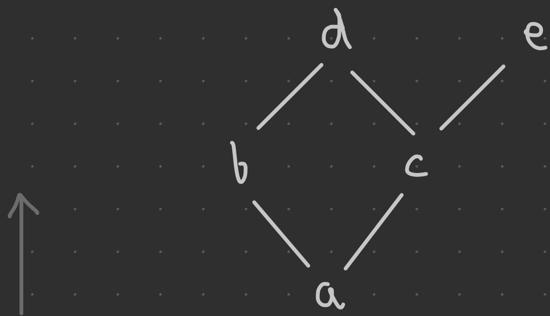
$(P ; \leq_P)$ $(Q ; \leq_Q)$

$f : P \to Q$

f preserve a $(\leq)$

$$p \leq_P p' \implies f\,p \leq_Q f\,p'$$

Diagramas Hasse



isso determina um poset!

$$a \prec b \overset{def}{\iff} a \le b \ \& \ \overbrace{(\forall t)[a \le t \le b \Rightarrow t = a \text{ ou } t = b]}^{(\forall a \le t \le b)[t = a \text{ ou } t = b]}$$

coberto por

outra notação: $a < b$

HW: corrija a def de $(\prec)$

Espaço métrico

espaço met. carrier métrica, dista

$$\mathcal{M} \equiv (M \,;\, d)$$

$$M \;:\; \mathrm{Set}\ \alpha$$

$$d \;:\; M \times M \longrightarrow \overbrace{\mathrm{Distance}}^{\mathbb{R}}$$

d-sym	$d(x,y) = d(y,x)$
d-triang	$d(x,y) \leq d(x,t) + d(t,y)$
d-pos	$d(x,y) \geq 0$
d-self	$d(x,x) = 0$ $\boxed{d(x,y) = 0 \Leftarrow x = y}$
d-zero	$d(x,y) = 0 \implies x = y$

Arr(Metr)

Quais são as setas

$$\mathcal{M} \xrightarrow{f} \mathcal{N}$$

$(M; d_M) \qquad (N; d_N)$

$f : M \longrightarrow N$

t.q. f é legal

$\Updownarrow$

?

N : Set ?

d_N : $N \times N \longrightarrow$ Dist

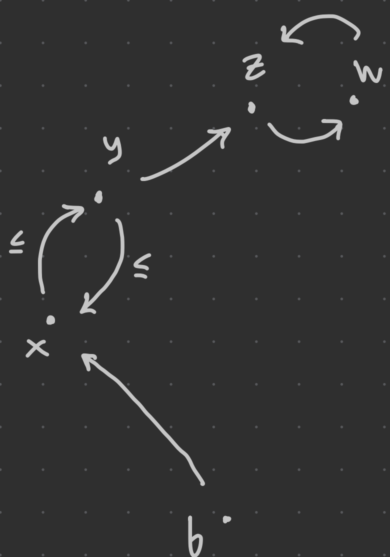
d_M : $M \times M \longrightarrow$ Dist

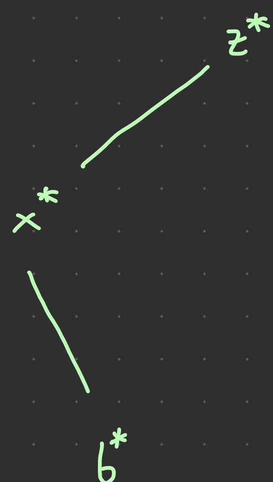
$\mathcal{M}$: Metric

θ. Justificar o nome (pré-ordem)

Proset $\rightsquigarrow$ Poset

$(P ; \leq)$ $\qquad$ $(P^* ; \leq^*)$





$$a \sim b \overset{\text{def}}{\iff} a \leq b \ \& \ b \leq a$$

$(\sim)$ é uma relação de equivalência

$(\sim)$-refl
$(\sim)$-trans
$(\sim)$-sym

portanto

$$P/_{\sim} \equiv \{ [a]_{\sim} \mid a \in P \}$$

$$P/_\sim \equiv \{ \quad [b] = \{b\}$$

$$[x] = \{x, y\} \qquad X \leq^* Y \overset{\text{def}}{\iff} (\overset{\forall ?}{\exists} x \in X)(\overset{\forall ?}{\exists} y \in Y)[\, x \leq y \,]$$

$$[y] = \{x, y\}$$

vs:

$$[z] = \{z, w\}$$

escolha $x \in X$

escolha $y \in Y$

$$[w] = \{w, z\} \qquad X \leq^* Y \underset{\text{def?}}{\iff} x \leq y$$

$$\}$$

θ. Justificar o nome pré-metrica.

Bolas

bola (aberta)

$$B_\varepsilon(c) \equiv \{ x \in M \mid d(x,c) < \varepsilon \}$$

$$B_\varepsilon[c] \equiv \{ x \in M \mid d(x,c) \leq \varepsilon \}$$

bola fechada

Limites

$(x_n \mid n : \mathbb{N}) \to \ell \iff (\forall \varepsilon > 0)[$ a $(x_n)_n$ eventualmente fica dentro da $B_\varepsilon(\ell)$ $]$

$(x_n)_{n \in \mathbb{N}}$