BING'S SPACE LIST

Space 1. The points of the space are the points of a straight line. Neighborhoods are the sets of points (strictly) between pairs of points. (This space is just the real line with the usual topology.)

Space 2. The points of the space are points of a horizontal straight line, and neighborhoods are *rays* extending to the right. Since rays contain their starting points, each of these neighborhoods has a leftmost point.

Space 3. Points are points of a horizontal line, and neighborhoods are *sects* (half-open intervals), open on the right. Each neighborhood has a leftmost point but no rightmost point.

Space 4. Points are points of the plane, and neighborhoods are interiors of circles. (This space is just the real plane witht he usual topology. Alternatively, it is the topological product of Space 1 with itself.)

Space 5. Points are points of the plane. Neighborhoods are formed from neighborhoods of Space 4 by discarding all irrational points (points whose coordinates are not both rational) except the centers of the circles. Thus a neighborhood consists of all rational points (points both of whose coordinates are rational) on the interior of a circle plus the center of the circle.

Space 6. Points are points of the plane. Neighborhoods are formed from neighborhoods of Space 4 by discarding all rational points except the centers of the circles. Notice that every neighborhood of Space 6 is uncountable, while every neighborhood of Space 5 is countable.

Space 7. The points of the space are the points of the plane. A neighborhood is a horizontally based rectangular "disk" with its top side and right-most side removed.

Space 8. Points are the points of the plane. Neighborhoods are of two sorts: (i) interiors of circles which miss the x-axis; (ii) interiors of circles minus the x-axis plus the centers of the circles.

Space 9. Points are the points of the plane on or above the x-axis. Neighborhoods are of two sorts: (i) interiors of circles which miss the x-axis; (ii) interiors of circles tangent to the x-axis from above plus the pointo of tangency.

Space 10. Points are the points of a horizontally based square disk in the plane. Its points are lexicographically ordered by their coordinates; that is. (x_1, y_1) precedes (x_2, y_2) iff either $x_1 < x_2$ or $x_1 = x_2$ and $y_1 < y_2$. Neighborhoods are of three types: (i) the set of points preceding some point; (ii) the set of points following some point; (iii) the set of points between two points (and not including those two).

Space 11. Points are elements of an ordered uncountable collection of points

$$p_1, p_2, p_3, \dots p_\omega, p_{\omega+1}, \dots p_{\omega 2}, p_{\omega 2+1}, \dots p_\alpha, \dots$$

such that each subset has a first element, and no element is preceded by uncountably many elements. Equivalently, take the points of the space to by the members of the first uncountable von Neumann ordinal ω_1 (also known as Ω). Neighborhoods are of three types: (i) the set of points preceding some point; (ii) the set of points following some point; (iii) the set of points between two points (and not including those two).

Space 12. Points are points on the plane. Neighboroods are finite point sets. This space is said to have the *discrete topology* since each point lies in a neighborhood containing only the point. Each point is *isolated*.

Space 13. A finite subspace of Space 4.

Space 14. A sequence of integers $n_1, n_2, n_3, \ldots$ is *rapidly diverging* iff

$$\lim_{i\to\infty} \frac{1}{n_i} = 0.$$

The points of the space are the natural numbers. Each point except 0 is isolated. A neighborhood of 0 is the union of $\{0\}$ and the complement of a rapidly diverging sequence.

Space 15. Points are points of the plane. Neighborhoods are of two types: (i) an open interval on a ray from the origin; (ii) the interior of a circle with its center at the origin.

Space 16. Points are points of the plane. Neighborhoods are of two types: (i) an open interval on a ray from the origin; (ii) the union of a family of open intervals, each containing the origin, such that each line through the origin contains one of the open intervals.

Space 17. Points are continuous real functions on $[0, 1]$. The distance between two points (functions) f, g is the maximum of $|f(x) - g(x)|$ for $x \in [0, 1]$. Neighborhoods are open balls with this metric.

Space 18. Points are continuous functions on $[0, 1]$ with codomain $\{0, 1\}$. (That is, points are characteristic functions of subsets of $[0, 1]$.) For each point g and each finite set of numbers $\{x_1, \ldots, x_n\}$ of $[0, 1]$, a neighborhood determinedby f and $x_1, \ldots, x_n$ is the set of all points of the space such that as functions they agree with f on $x_1, \ldots, x_n$. Notation:

$$N(f; x_1, \ldots, x_n) = \{\, g : [0, 1] \to \{0, 1\} \;\mid\; g(x_i) = g(x_i) \text{ for all } i \in \{1, \ldots, n\} \,\}$$

Space 19. Points are functions from $\mathbb{Z}$ to $\{0, 1\}$. Neighborhoods are as in Space 18.

Space 20. The subspace of Space 18 obtained by deleting all points (functions) which take on uncountably many 0's. (Thus Space 20 consists of all characteristic functions of countable subsets of $[0, 1]$ with the subspace topology induced by Space 18.)

Space 21. The subspace of the real line whose points are the points of the Cantor set.

Space 22. Points of the space are the rational points of the plane on and above the x-axis. Neighborhoods are of two types: (i) an open interval on the x-acis (containing, remember, only rational points); (ii) if p, q, r are the vertices of an equilateral triangle with p a rational point above the x-axis and q, r points on the x-axis (not in the space), the union of $\{p\}$ open intervals on the x-axis about q and r is a neighborhood. (Naturally, these open intervals about q and r do not contain q and r, since the space contains only rational points.)

Space 23. The subspace of the plane obtained from the closure of the graph of $y = \sin\frac{1}{x}$.

Space 24. The subspace of the plane obtained from the set which is the union of closed intervals $A_\infty, A_1, A_2, \ldots$ where A_∞ is an interval from $(0,0)$ to $(1,0)$, and A_n is an interval from $(0,0)$ to $(1,1/n)$. of $y = \sin\frac{1}{x}$.

Space 25. Let $X_1, X_2, X_3, \ldots$ be the sets obtained by shrinking and translating the set used in Space 24 so that: the base of X_1 is the closed interval from $(0,0)$ to $(1/2,0)$; the base of X_2 is the closed interval from $(1/2,0)$ to $(3/4,0)$; the base of X_3 is the closed interval from $(3/4,0)$ to $(7/8,0)$; ...etc. Space 25 is the subspace of the plane which is the union of the closed interval from $(0,0)$ to $(1,0)$ with $\bigcup_{i=1}^{\infty} X_i$.

Space 26. Let Δ be a Cantor set on the x-axis (say the one obtained from the closed interval from $(0,0)$ to $(1,0)$ by removing middle thirds), and let X be the union of all the straight line segments joining the points of Δ to a point (say the point $(1/2,1/2)$) off the x-axis. (X is thus a "cone over a Cantor set".) We call $(1/2,1/2)$ the *vertex* of X. Now let Y be a subset of X such that: (a) Y contains the vertex of X; (b) no radius of X (i.e., no line segment joining the vertex to a point of Δ) contains more than two points of Y; (c) Y intersects each closed subset of X that intersects uncountably many radii of X. (The existence of such a set Y is guaranteed by the Well-Ordering Theorem.) Space 26 is the subspace of the plane obtained from Y.