

---

Nome:

---

26/06/2019

**Regras:**

- I. Não vires esta página antes do começo da prova.
- II. Nenhuma consulta de qualquer forma.
- III. Nenhum aparelho ligado (por exemplo: celular, tablet, notebook, *etc.*).<sup>1</sup>
- IV. Nenhuma comunicação de qualquer forma e para qualquer motivo.
- V.  $\forall x(\text{Colar}(x) \rightarrow \neg \text{Passar}(x, \text{TOPO}))$ .
- VI. Use caneta para tuas respostas.
- VII. Responda dentro das caixas indicadas.
- VIII. Escreva teu nome em *cada* folha de rascunho extra *antes de usá-la*.
- IX. Entregue *todas* as folhas de rascunho extra, juntas com tua prova.
- X. Nenhuma prova será aceita depois do fim do tempo!
- XI. Os pontos bônus são considerados apenas para quem consiga passar sem.<sup>2</sup>
- XII. **Responda em 1 vogal e 1 consoante.**<sup>3</sup>

*Boas provas!*

---

<sup>1</sup>Ou seja, *desligue antes* da prova.

<sup>2</sup>Por exemplo, 25 pontos bônus podem aumentar uma nota de 5,2 para 7,7 ou de 9,2 para 10,0, mas de 4,9 nem para 7,4 nem para 5,0. A 4,9 ficaria 4,9 mesmo.

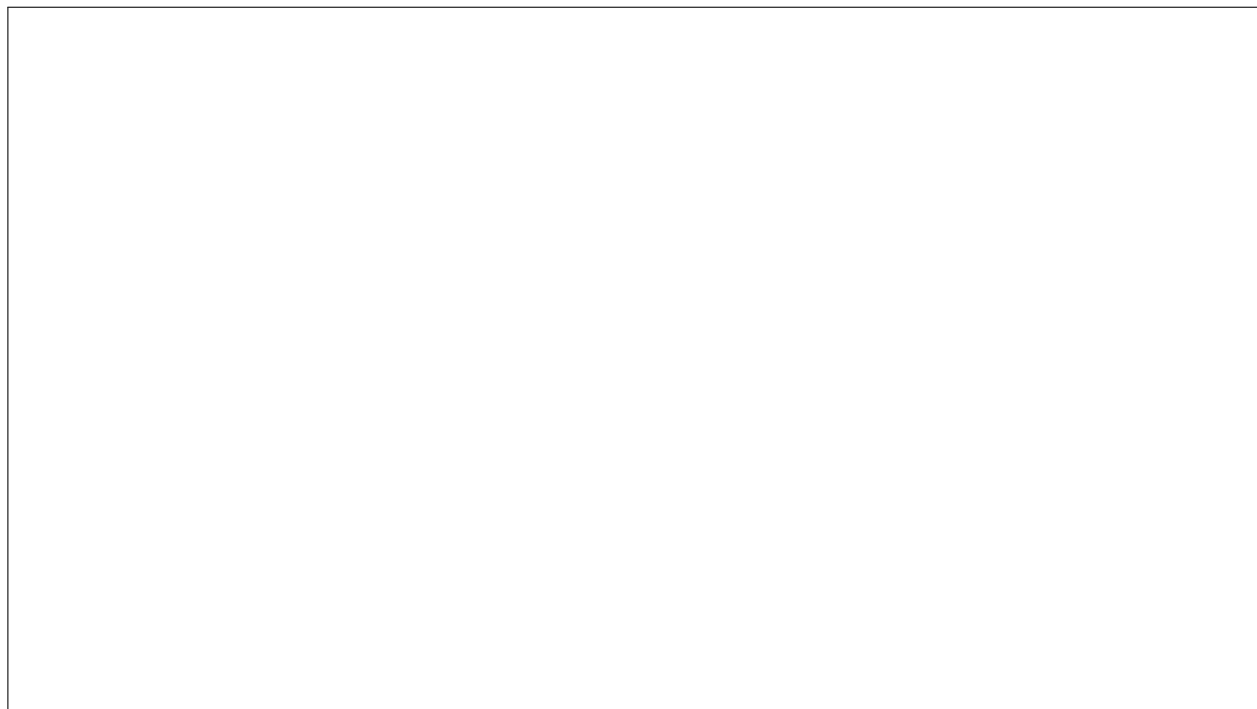
<sup>3</sup>Provas com respostas em mais que isso não serão corrigidas (tirão 0 pontos).

(24) **A**

Prove that one of the following is a topological space and that the other one is not.

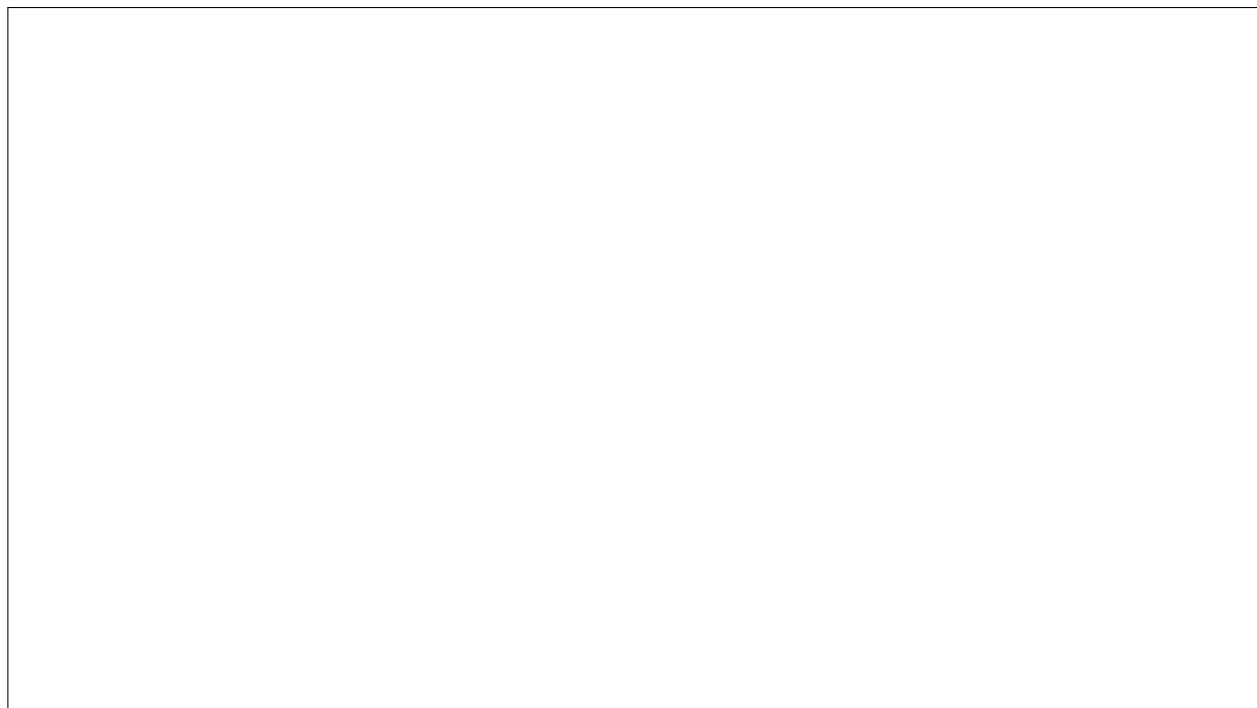
**A1.**  $X$  is an uncountable set;  $\mathcal{O}$  is the collection of all sets that are countable or  $X$  itself.

PROOF/REFUTATION.



**A2.**  $X$  is an uncountable set;  $\mathcal{O}$  is the collection of all sets that are cocountable or empty.

PROOF/REFUTATION.



(42) **E**

$X$  is  $(0, 1)$ ;  $\mathcal{O}$  is the set of all intervals of the form  $(0, 1 - \frac{1}{n})$ , for  $n \in \mathbb{N}_{>0}$ .

(10) **E1.** Is this space  $T_0$ ?

ANSWER: \_\_\_\_\_ .

(10) **E2.** Is this space  $T_3$ ?

ANSWER: \_\_\_\_\_ .

(10) **E3.** Is this space  $T_4$ ?

ANSWER: \_\_\_\_\_ .

(12) **E4.** Which of this space's open sets are compact?

ANSWER:

(52) **O**

$X$  is  $[-1, 1]$ ;  $\mathcal{O}$  is the topology generated by the basis of all the sets of the form  $[-1, b)$  for  $b > 0$  and all the sets of the form  $(a, 1]$  for  $a < 0$ .

(8) **O1.** This space is  $T_0$ .

PROOF.

(11) **O2.** Is this space  $T_1$ ?

ANSWER: \_\_\_\_\_ .

(11) **O3.** Is this space  $T_4$ ?

ANSWER: \_\_\_\_\_ .

(11) **O4.** Is this space compact?

ANSWER: \_\_\_\_\_ .

(11) **O5.** Is this space second-countable?

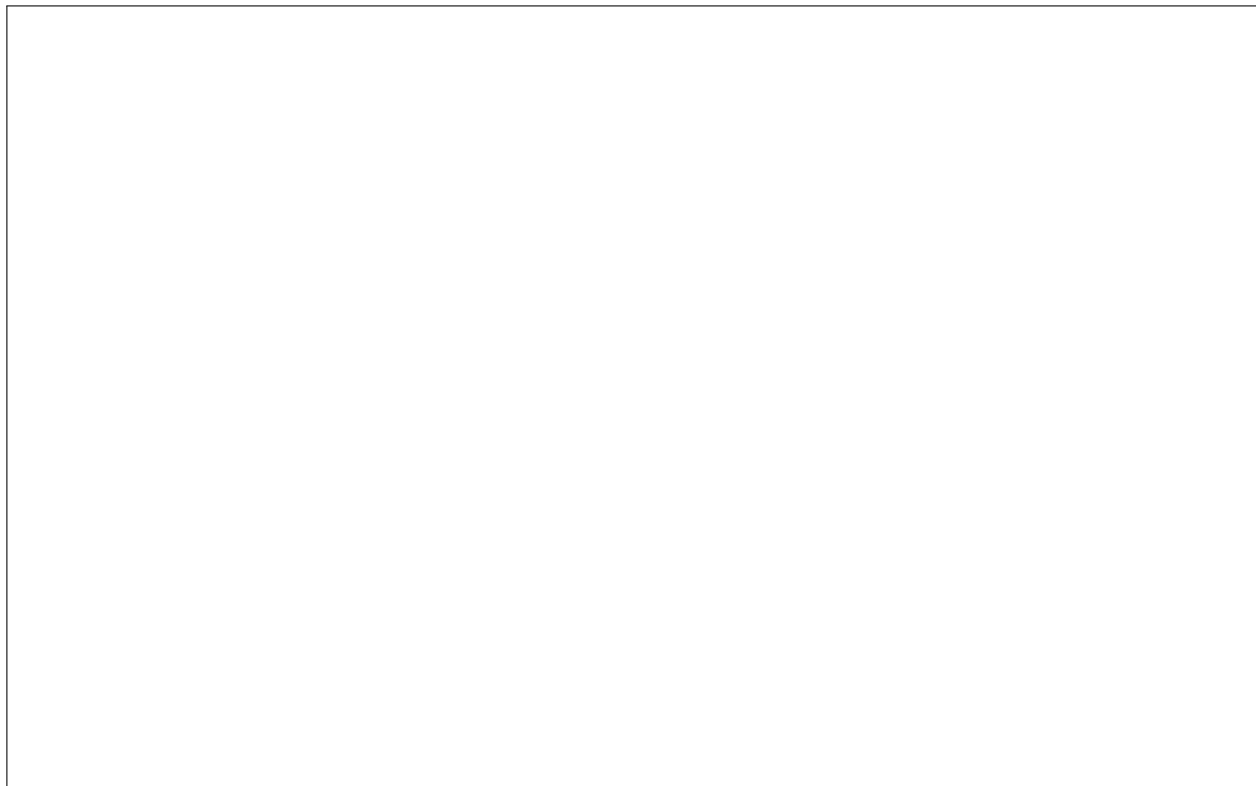
ANSWER: \_\_\_\_\_ .

(36) **B**

**Proposition.** *Let  $X$  be a space and let  $(x_n)_n$  be a sequence of points of  $X$ . Then  $(x_n)_n$  converges to at most one limit.*

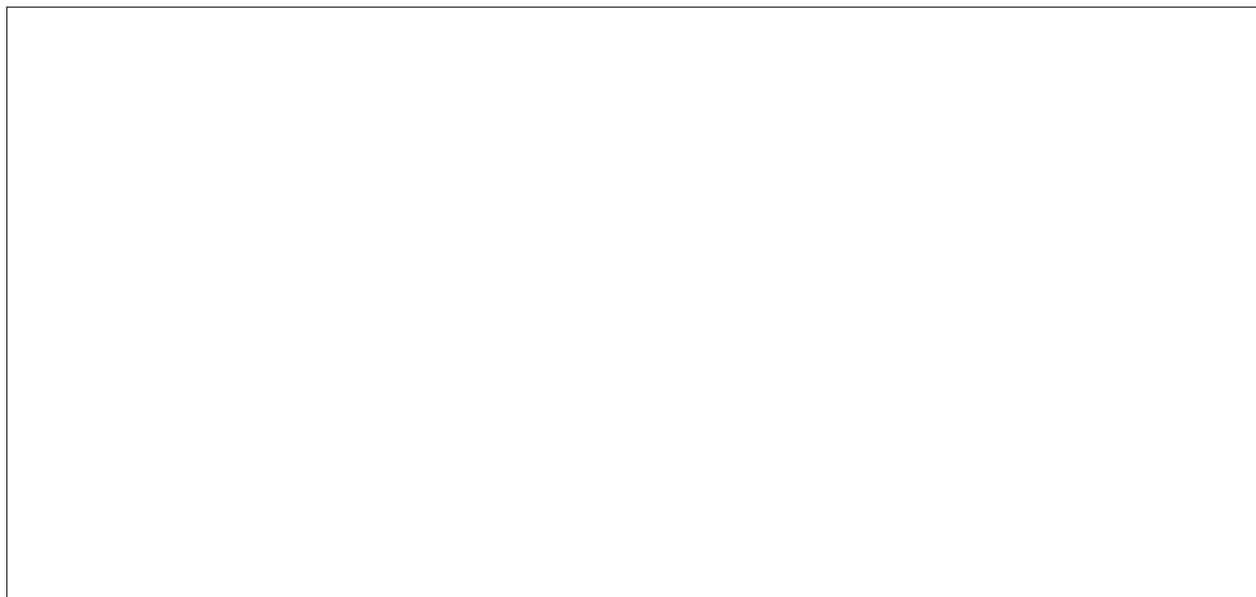
(24) **B1.** Show that the Proposition is true for metric spaces.

PROOF.



(12) **B2.** Is the Proposition true for topological spaces?

ANSWER: \_\_\_\_\_ .



(48) **C**

**Proposition.** *Let  $X$  be a space and let  $S \subseteq X$ . Then every point of  $\overline{S}$  is the limit of some sequence of points of  $X$ .*

(24) **C1.** Show that the Proposition is true for metric spaces.

PROOF.

(24) **C2.** Show that the Proposition may fail for topological spaces.

*Dica: Some vowel space might help.*

COUNTEREXAMPLE.

Só isso mesmo.

## RASCUNHO

## RASCUNHO



## RASCUNHO