Tópicos: Semântica Denotacional, 2023.2
(Turma T12 do Thanos)

Prova 1.1
(points: 100; bonus: $0^\flat$; time: 64')

Nome:

2023-12-20

Regras:

I. Não vires esta página antes do começo da prova.

II. Nenhuma consulta de qualquer forma.

III. Nenhum aparelho ligado (por exemplo: celular, tablet, notebook, *etc.*).[1]

IV. Nenhuma comunicação de qualquer forma.

V. $(\forall x)\,[\,\mathrm{Colar}(x) \implies \neg\mathrm{Passar}(x, \mathrm{SEMANTICS})\,]$.[2]

VI. Responda dentro das caixas indicadas.

VII. Nenhuma prova será aceita depois do fim do tempo—mesmo se for atraso de 1 segundo.

VIII. **Escolha o D e até 2 dos I, M, L.**[3]

Esclarecimento: Relaxe.

Boas provas!

[1] Ou seja, *desligue antes* da prova.
[2] Se essa regra não faz sentido, melhor desistir desde já.
[3] Provas violando essa regra (com respostas em mais problemas) não serão corrigidas (tirarão 0 pontos).

Definitions

Def (directed). Let P be a poset and $D \subseteq P$.

$$D \text{ directed} \iff \begin{cases} D \text{ is inhabited} \\ (\forall d_1, d_2 \in D)\,(\exists d \in D)\,[\, d \geq \{d_1, d_2\}\,] \end{cases}$$

Def (dcpo). A poset P is a *dcpo* iff every directed subset of P has a lub (in P).

Def (Scott-open). A subset $S \subseteq P$ of an inductive poset is *Scott-open* if (1) S is upwards-closed; (2) for every inhabited chain $C \subseteq P$,

$$\sup C \in S \implies (\exists c \in C)\,[\, c \in S\,]\,.$$

Def (countably continuous). A monotone mapping $\pi : P \to Q$ from an inductive poset to another is *countably continuous* iff for every inhabited countable chain $C \subseteq P$,

$$\pi(\sup C) = \sup \pi(C).$$

Def (continuous, compact). A mapping $\pi : (A \rightharpoonup E) \to (B \rightharpoonup M)$ is *continuous* iff it is monotone and *compact:*

$$(\forall f : A \rightharpoonup E)(\forall b \in B)\,(\forall m \in M)\,[\, \pi\ f\ b = m \implies (\exists f_0 \subseteq f)\,[\, f_0 \text{ is finite } \ \&\ \ \pi\ f_0\ b = m\,]\,]$$

Def (inflating). Let P be a poset and let $f : P \to P$ be an endomap on P.

$$f \text{ inflating} \iff (\forall x \in P)\,[\, x \leq f\ x\,]\,.$$

Def. Let L be a complete lattice and $T : L \to L$ be monotonic. We define by transfinite recursion:

$$\begin{aligned} T \uparrow 0 &\stackrel{\text{def}}{=} \bot \\ T \uparrow (\alpha^+) &\stackrel{\text{def}}{=} T(T \uparrow \alpha) \\ T \uparrow \lambda &\stackrel{\text{def}}{=} \text{lub}\,\{\, T \uparrow \alpha \mid \alpha < \lambda \,\}\,. \end{aligned}$$

D

Let L be a dcpo with $\bot$. Let

$$\mathcal{L} \stackrel{\text{def}}{=} \{\, f : L \to L \mid f \text{ monotone and inflating} \,\}\,.$$

and order $\mathcal{L}$ with the pointwise ordering. (i) $\mathcal{L}$ is inhabited; (ii) $\mathcal{L}$ is directed.
Demonstração.

I

Define the semantics of **while** and prove: For any $B : \mathsf{exp}[\mathsf{bool}]$ and $C : \mathsf{comm}$,

$$[\![\textbf{while } B \textbf{ do } C]\!] = [\![\textbf{if } B \textbf{ then } (C; \textbf{while } B \textbf{ do } C)]\!].$$

Resposta.

M

*Escolha até **duas**.*

M1. Every countably continuous monotone mapping $\pi : P \to P$ on an inductive poset has exactly one strongly least fixpoint x^*.

Let $\pi : (A \rightharpoonup E) \to (B \rightharpoonup M)$.

$$\underbrace{\pi \text{ continuous}}$$

$$\textbf{M2} \Downarrow \quad \Uparrow \textbf{M3}$$

$$\overbrace{(\forall f : A \rightharpoonup E)(\forall b \in B)\,(\forall m \in M)\,[\,\pi\ f\ b = m \iff (\exists f_0 \subseteq f)\,[\,f_0 \text{ is finite } \ \&\ \ \pi\ f_0\ b = m\,]\,]}$$

M4. If $S \subseteq (A \rightharpoonup E)$ is a non-empty chain in a partial function poset and $f_0 \subseteq \sup S$ is a finite function, then there exists some $g \in S$ such that $f_0 \subseteq g$.

M5. The family of Scott open subsets of an inductive poset is a topology.

Demonstração da _____ .

Demonstração da _____ .

L

*Escolha até **duas**.*

Let L be a complete lattice and $T : L \to L$ be a monotonic endomap.

L1. T has a least fixpoint $\mathrm{lfp}(T)$. Furthermore,

$$\mathrm{lfp}(T) = \mathrm{glb}\,\{\, x \mid T(x) = x \,\} = \mathrm{glb}\,\{\, x \mid T(x) \leq x \,\}\,.$$

L2. For all α, $T \uparrow \alpha \leq \mathrm{lfp}(T)$.

L3. For all α, $T \uparrow \alpha \leq T \uparrow (\alpha + 1)$.

L4. For all α, β, $\alpha < \beta \implies T \uparrow \alpha \leq T \uparrow \beta$.

Demonstração da _____ .

Demonstração da _____ .

Só isso mesmo.

Rascunho